Examination Number:

Set:

Shore

Year 12

HSC Assessment Task 5 - Trial HSC

14th August 2015

Mathematics Extension 1

General Instructions

- Reading time − 5 minutes
- Working time − 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this question paper
- Answer Questions 1–10 on the Multiple Choice Answer Sheet provided
- Start each of Questions 11–14 in a new writing booklet
- In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.
- Write your examination number on the front cover of each booklet
- If you do not attempt a question, submit a blank booklet marked your examination number and "N/A" on the front cover

Total marks – 70

Section I Pages 3–6

10 marks

- Attempt Questions 1−10
- Allow about 15 minutes for this section

Section II Pages 7–12

60 marks

- Attempt Questions 11−14
- Allow about 1 hour 45 minutes for this section

Note: Any time you have remaining should be spent revising your answers.

DO NOT REMOVE THIS PAPER FROM THE EXAMINATION ROOM

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the Multiple Choice Answer Sheet for Questions 1–10.

1 What is the value of $\displaystyle\int_0^{\frac{1}{5}} \frac{1}{\sqrt{1-25x^2}}\,dx$?

(A) $\dfrac{\pi}{10}$

(B) $\dfrac{\pi}{5}$

(C) $\dfrac{\pi}{2}$

(D) $\dfrac{\pi}{25}$

2 Which of the following is a simplification of $4\log_e \sqrt{e^x}$?

(A) $4\sqrt{x}$

(B) $\dfrac{1}{2}x$

(C) $2x$

(D) x^2

3 The acute angle between the lines $2x - y = 0$ and $kx - y = 0$ is equal to $\dfrac{\pi}{4}$.

What is the value of k?

(A) $k = -3$ or $k = -\dfrac{1}{3}$

(B) $k = -3$ or $k = \dfrac{1}{3}$

(C) $k = 3$ or $k = -\dfrac{1}{3}$

(D) $k = 3$ or $k = \dfrac{1}{3}$

4 What is the domain and range of $y = 2\cos^{-1}(x-1)$?

(A) Domain : $0 \le x \le 2$. Range: $0 \le y \le \pi$

(B) Domain : $-1 \le x \le 1$. Range: $0 \le y \le \pi$

(C) Domain : $0 \le x \le 2$. Range: $0 \le y \le 2\pi$

(D) Domain : $-1 \le x \le 1$. Range: $0 \le y \le 2\pi$

5 Which of the following is a simplification of $\dfrac{1-\cos 2x}{\sin 2x}$?

(A) $1-\cot x$

(B) 1

(C) $\cot x$

(D) $\tan x$

6 The domain of the function $y = x^2(x-2)^2$ must be restricted to have an inverse function.

Which of the following restrictions on the domain allows for an inverse function to exist?

(A) $x \ge 2$

(B) $0 \le x \le 2$

(C) $x \ge 0$

(D) $x \le 2$

7 A particle is moving in simple harmonic motion with displacement x. The velocity v of the particle is given by

$$v^2 = 4(25 - x^2).$$

What is the amplitude a of the motion and the maximum speed of the particle?

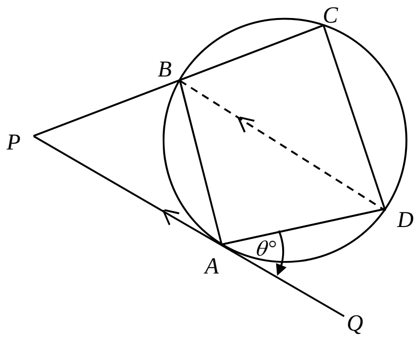
(A) $a = 2$ and maximum speed is 5 m/s

(B) $a = 2$ and maximum speed is 10 m/s

(C) $a = 5$ and maximum speed is 5 m/s

(D) $a = 5$ and maximum speed is 10 m/s

8 In the diagram $ABCD$ is a cyclic quadrilateral. The tangent PQ touches the circle at A. The diagonal BD is parallel to the tangent PQ. QA produced intersects with CB at P. Let $\angle QAD = \theta°$



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What is the size of $\angle BCD$?

(A) $\theta°$

(B) $2\theta°$

(C) $(\pi - \theta)°$

(D) $(\pi - 2\theta)°$

9 Which of the following is the general solution of $2\sin\frac{x}{2} = \sqrt{3}$?

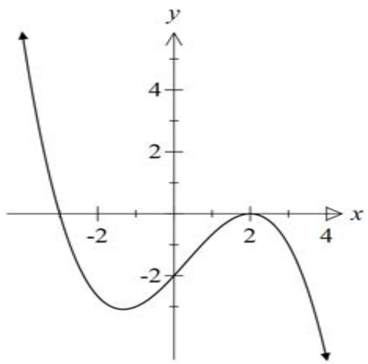
(A) $x = 4n\pi \pm \frac{\pi}{3}$, where n is an integer.

(B) $x = 2n\pi + \frac{\pi}{3}$, where n is an integer.

(C) $x = 2n\pi + (-1)^n \times \frac{2\pi}{3}$, where n is an integer.

(D) $x = 4n\pi + (-1)^n \times \frac{2\pi}{3}$, where n is an integer.

10 The polynomial graph shown below has equation $y = A(x + B)(x + C)^2$.



Which of the following is true?

(A) $A = -\frac{1}{6}, B = 3, C = -2$

(B) $A = \frac{1}{6}, B = -3, C = 2$

(C) $A = 1, B = 3, C = -2$

(D) $A = -1, B = 3, C = -2$

End of Section I

Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

(a) Solve the inequality $\dfrac{x-3}{x+4} \le 2$. **3**

(b) The point $C(-3,8)$ divides the interval AB externally in the ratio k : 1. Find the value of k if A is the point $(6,-4)$ and B is the point $(0,4)$. **2**

(c) Find $\dfrac{d}{dx}\left(x\sin^{-1}2x\right)$. **2**

(d) Use the substitution $u = \ln x$ to evaluate $\displaystyle\int_{\frac{1}{e}}^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}}$. **3**

(e) The region bounded by the curve $y = \cos 3x$ and the x axis between the lines $x = 0$ and $x = \dfrac{\pi}{6}$ is rotated through one complete revolution about the x-axis. Find the exact volume of the solid formed. **3**

(f) Use one application of Newton's method with an initial approximation of $x = 1.5$ to find the next approximation to the root of the equation $2\log_e x - \cos x = 0$. Give your answer correct to 2 significant figures. **2**

Question 12 (15 marks) Use a SEPARATE writing booklet

(a) Find the constant term of the expansion $\left(\dfrac{1}{x^2} - 2x^3\right)^{10}$. **3**

(b) The polynomial $P(x) = x^3 + ax^2 + bx - 6$ has a remainder of 8 when divided by $(x+1)$ and $(x-3)$ is a factor of the polynomial $P(x)$. Find the values of a and b. **3**

(c) The displacement x metres of a particle moving in simple harmonic motion is given by

$$x = 4\cos \pi t$$

where time t is in seconds.

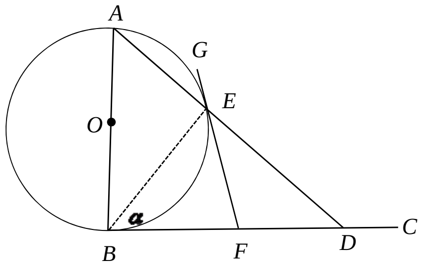
(i) What is the period of the oscillation? **1**

(ii) What is the speed of the particle as it moves through the equilibrium position? **1**

Question 12 continues on the following page

Question 12 (continued)

(d) In the diagram AB is a diameter of the circle centre, O, and BC is tangential to the circle at B. The line AED intersects the circle at E and BC at D. The tangent to the circle at E intersects BC at F. Let $\angle EBF = \alpha$.



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Copy or trace the diagram into your writing booklet.

(i) Prove that $\angle FED = \frac{\pi}{2} - \alpha$. **2**

(ii) Prove that $BF = FD$. **2**

(e) Use mathematical induction to prove that $5^n + 2 \times 11^n$ is a multiple of 3 for all integers $n \geq 1$. **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

(a) An oven which had been heated to $180°C$ was switched off when the cook was finished baking at 11:30 am. The oven was in a kitchen which was kept at a constant temperature of $22°C$.

After t minutes, the temperature, $T°C$, of the oven was given by:

$$T = 22 + Be^{-kt}$$

where A, B and k are positive constants.

(i) If after 10 minutes, the oven's temperature has dropped to $115°C$, find B and the exact value of k . **2**

(ii) At what time will the oven's temperature drop to $23°C$. Give your answer correct to the nearest minute. **2**

(b) A particle is moving along the x-axis so that its acceleration after t seconds is given by

$$\ddot{x} = -e^{-\frac{x}{2}}$$

The particle starts at the origin with an initial velocity of 2 m/s.

(i) If v is the velocity of the particle, find v^2 as a function of x. **2**

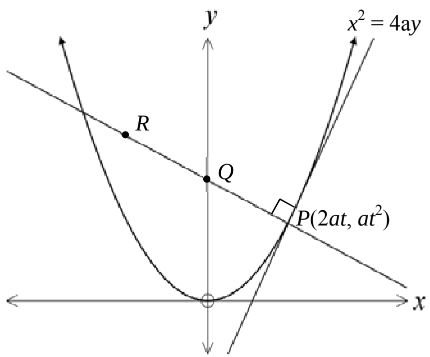
(ii) Given that $v > 0$ throughout the motion, show that the displacement x as a function of time t is given by **3**

$$x = 4\log_e\left(\frac{t+2}{2}\right)$$

Question 13 continues on the following page

Question 13 (continued)

(c)



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The diagram shows the parabola $x^2 = 4ay$. The normal at the point $P(2at, at^2)$ cuts the y-axis at Q and is produced to a point R on the normal such that $PQ = QR$.

(i) Show that the equation of the normal at P is given by **2**

$$x + ty = 2at + at^3.$$

(ii) Find the coordinates of Q and show that the coordinates of R are $(-2at,\ at^2 + 4a)$. **2**

(iii) Show that the locus of R is another parabola and state its vertex. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

(a) A particle is projected under gravity g with speed V metres per second at an angle of θ from a point O on horizontal ground. It strikes the ground at P, where $OP = R$.

The equations of motion of the particle are

$$x = Vt\cos\theta \text{ and}$$
$$y = Vt\sin\theta - \frac{1}{2}gt^2 \qquad \text{(Do not derive these equations)}$$

(i) If θ is $45°$ show that the equation of trajectory of the particle is given by **2**

$$y = x - g\frac{x^2}{V^2}$$

(ii) Hence, or otherwise, show that $R = \dfrac{V^2}{g}$. **2**

(iii) A bullet is fired from O with velocity 30 m/s at an angle of $45°$ to the horizontal. Find the speed of the ball when it has travelled a horizontal distance of 15 m from its starting point. (Take $g = 10\,\text{ms}^{-2}$). Give your answer correct to 1 decimal place. **3**

(b) Initially, a ladder leaning against a wall just reaches a window sill 2.4 metres above the ground. The foot of the ladder is x metres from the wall and it makes an angle of θ (in radians) with the horizontal. The foot of the ladder is slipping at a rate of 2 cm/s.

(i) Show that $\dfrac{dx}{d\theta} = -\dfrac{2.4}{\sin^2\theta}$. **2**

(ii) Find the rate of change of the angle when $\theta = \dfrac{\pi}{4}$. **2**

(c)

(i) Use the binomial theorem to give the expansion of $(1+x)^{2n}$. **1**

(ii) Hence prove the following identity. **3**

$${}^{2n}C_1 + 3\,{}^{2n}C_3 + \ldots\ldots + (2n-1)\,{}^{2n}C_{2n-1} = 2\,{}^{2n}C_2 + 4\,{}^{2n}C_4 + \ldots\ldots + 2n\,{}^{2n}C_{2n}$$

END OF PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \,, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

Note $\ln x = \log_e x, \quad x > 0$

SHORE TRIAL EXAM 2015

Mathematics Extension 1

Section I Multiple choice

1. $\int_0^{\frac{1}{5}} \frac{1}{\sqrt{1-25x^2}}\,dx = \int \frac{1}{\sqrt{25(\frac{1}{25}-x^2)}}\,dx$

$= \frac{1}{5}\left[\sin^{-1}5x\right]_0^{\frac{1}{5}}$

$= \frac{1}{5}\left[\sin^{-1}1 - \sin^{-1}0\right]$

$= \frac{1}{5}\left[\frac{\pi}{2} - 0\right]$

$= \underline{\underline{\frac{\pi}{10}}}$ (A)

2. $4\log_e\sqrt{e^x} = \log\left((e^x)^{\frac{1}{2}}\right)^4$

$= \log e^{2x}$

$= \underline{\underline{2x}}$ (C)

3. $m_1 = k \quad m_2 = 2 \quad \theta = \frac{\pi}{4}$

$\tan\theta = \left|\frac{m_2-m_1}{1+m_1m_2}\right|$

$\tan\frac{\pi}{4} = \left|\frac{2-k}{1+2k}\right|$

$\pm 1 = \frac{2-k}{1+2k}$

$2-k = 1+2k$ or $2-k = -1-2k$

$1 = 3k \qquad \underline{\underline{k=-3}}$ (B)

$\underline{\underline{\frac{1}{3} = k}}$ or

4. D: $-1 \le x-1 \le 1$

$\underline{\underline{0 \le x \le 2}}$

R: $0 \le \frac{y}{2} \le \pi$ (C)

$\underline{\underline{\text{or } y \le 2\pi}}$

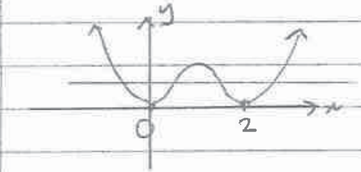
5. $\frac{1-\cos 2x}{\sin 2x} = \frac{1-(\cos^2x-\sin^2x)}{2\sin x\cos x}$

$= \frac{(1-\cos^2x)+\sin^2x}{2\sin x \cos x}$

$= \frac{\cancel{2}\sin^2x}{\cancel{2}\sin x\cos x}$

$= \frac{\sin x}{\cos x}$

$= \underline{\underline{\tan x}}$ (D)

6. 

$x \ge 2$ (A)

7. $v^2 = 4(25-x^2)$

$v^2 = n^2(a^2-x^2)$

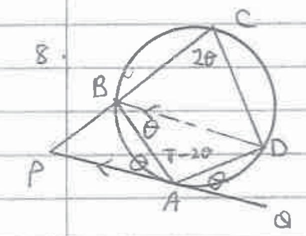
$\therefore\ \underline{a=5}$

Max speed when $x=0$

$\therefore\ v^2 = 4\times 25$

$v = \pm 10$

$\underline{\text{speed} = 10\ \text{m/s}}$ (D)

8. 

(B)

9. $\sin\frac{x}{2} = \frac{\sqrt{3}}{2}$

$\frac{x}{2} = n\pi \pm (-1)^n\sin^{-1}\frac{\sqrt{3}}{2}$

$= n\pi \pm (-1)^n\frac{\pi}{3}$

$x = 2n\pi \pm (-1)^n\frac{2\pi}{3}$ (C)

10. Polynomial : $a<0$ y intercept -2

A. $-\frac{1}{6}(x+3)(x-2)^2$ y intercept -2 $a<0$ ✓

B. $+\frac{1}{6}(x+3)(x+2)^2$ " " -2 $a>0$ (A)

c. $1\ (x-3)(x+2)^2$ " " -12 ×

d. $-1\ (x-3)(x+2)^2$ " 12 ×

SECTION II

Marker's Comments

Question 11:

a) $\frac{x-3}{x+4} \leq 2$

critical values: $x \neq -4$

Solve $x-3 = 2x+8$

$-11 = x$

test $x = 0$ $\quad -\frac{3}{4} \leq 2$ T.

$x \leq -11$ or $x > -4$

OR $x\,(x+4)^2$ $\quad \frac{x-3}{x+4}(x+4)^2 \leq 2(x+4)^2$

$x \neq -4$

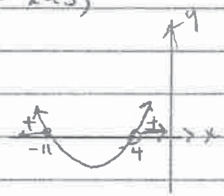
$(x-3)(x+4) \leq 2(x+4)^2$

$0 \leq 2(x+4)^2 - (x-3)(x+4)$

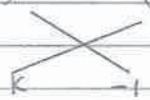
$0 \leq (x+4)[2x+8 - x+3)$

$0 \leq (x+4)(x+11)$

$\therefore$ $x \leq -11$ or $x > -4$



b) $C(-3, 8)$ $\quad$ A $(6,-4)$ $\quad$ B $(0,4)$



$\therefore\ -3 = \frac{0-6}{k-1}$

$-3k+3 = -6$

$9 = 3k$

$k = 3$

Marker's Comments

c) $\frac{d}{dx}(x \sin^{-1} 2x) = x \cdot \frac{2}{\sqrt{1-4x^2}} + \sin^{-1} 2x \times 1$

$= \frac{2x}{\sqrt{1-4x^2}} + \sin^{-1} 2x$

d) $u = \ln x$ $\qquad x = \sqrt{e}$ $\quad u = \frac{1}{2}$

$\frac{du}{dx} = \frac{1}{x}$ $\qquad x = \frac{1}{e}$ $\quad u = \ln e^{-1}$

$du = \frac{dx}{x}$ $\qquad u = -1$

$$\int_{\frac{1}{e}}^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}} = \int_{-1}^{\frac{1}{2}} \frac{du}{\sqrt{1-u^2}}$$

$= \left[\sin^{-1} u\right]_{-1}^{\frac{1}{2}}$

$= \sin^{-1}\frac{1}{2} - \sin^{-1}(-1)$

$= \frac{\pi}{6} + \frac{\pi}{2}$

$= \frac{4\pi}{6}$

$= \frac{2\pi}{3}$

e) $y = \cos 3x$

$$V_{ol} = \pi \int_0^{\frac{\pi}{6}} \cos^2 3x \, dx$$

$= \pi \int_0^{\pi/6} \left[\frac{1}{2} + \frac{1}{2}\cos 6x\right] dx$

$= \frac{\pi}{2}\left[x + \frac{1}{6}\sin 6x\right]_0^{\frac{\pi}{6}}$

$= \frac{\pi}{2}\left[\left(\frac{\pi}{6} + \frac{1}{6}\sin \pi\right) - (0-0)\right]$

$= \frac{\pi}{2} \times \frac{\pi}{6}$

$= \frac{\pi^2}{12}$ units^3

Marker's Comments

(f) let $f(x) = 2\ln x - \cos x$

$f(1.5) = 2\ln 1.5 - \cos 1.5$

$= 0.740193\ldots$ (A)

$f'(x) = \frac{2}{x} + \sin x$

$f'(1.5) = \frac{2}{1.5} + \sin 1.5$

$= 2.3308\ldots$ (B)

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5 - \frac{0.74\ldots}{2.33\ldots}$

$= 1.1824\ldots$

$= \underline{\underline{1.2}}$ (2 sig figures)

Question 12:

Marker's Comments.

a) $\left(\frac{1}{x^2} - 2x^3\right)^{10}$

General term is ${}^{10}C_k (x^{-2})^{10-k} (-2)^k (x^3)^k$ — 1 mark

Constant term is the coefficient of x^0

$\therefore x^0 = x^{-20+2k+3k}$

$0 = 5k - 20$

$4 = k$ — 1 mark

$\therefore$ Constant term is $\underline{\underline{{}^{10}C_4 (-2)^4 = 3360}}$ — 1 mark [3]

(b) $P(x) = x^3 + ax^2 + bx - 6$ $\quad P(3) = 0$ and $P(-1) = 8$

$P(3) = 3^3 + a \cdot 3^2 + 3b - 6$

$0 = 21 + 9a + 3b \quad (\div 3)$

$\underline{-7 = 3a + b}$ ① — 1 mark

$P(-1) = -1 + a - b - 6$

$8 = a - b - 7$

$\underline{15 = a - b}$ ② — 1 mark

① + ② $\quad 8 = 4a$

$2 = a$ $\quad$ sub into ①

$-7 = 6 + b$

$-13 = b$

$\underline{\underline{\therefore a = 2 \quad b = -13}}$ — 1 mark [3]

(c) $x = 4\cos \pi t$

(i) $P = \frac{2\pi}{\pi} = 2$ — 1 mark

(ii) $\dot{x} = -4\pi \sin \pi t$.

$x = 0$ when
$\frac{\pi}{2} = \pi t$
$\frac{1}{2} = t$

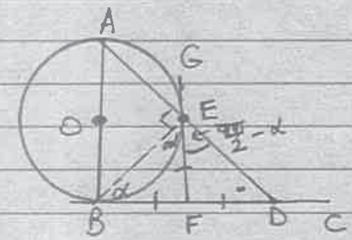
When $t = \frac{1}{2}$

Poorly done.

$\dot{x} = -4\pi$

Speed $= 4\pi$ m/s. — 1 mark [2]

(d)



(i) $BF = FE$
(tangents drawn from an external pt)

$\therefore \angle BFF = \alpha$ — 1 mark
($\angle$s opposite equal sides equal)

$\angle AEB = \frac{\pi}{2}$ (angle in a semicircle)

$\therefore \angle FED = \frac{\pi}{2} - \alpha$ (straight $\angle$) — 1 mark [2]

(ii) $\angle EDF = \frac{\pi}{2} - \alpha$ (angle sum of ΔBED)

$\therefore \Delta FED$ is isosceles

$\therefore FE = FD$ — 1 mark

Since $BF = FE$
and $FE = FD$
then $BF = FD$ — 1 mark [2]

e) Step 1: Prove true for $n = 1$

$5^1 + 2 \times 11 = 5 + 22$
$= 27$ which is a multiple of 3. — 1 mark

Step 2: Assume true for $n = k$

$5^k + 2 \times 11^k = 3M$ where M is an integer

ie $5^k = 3M - 2 \times 11^k$. — 1 mark

Now prove true for $n = k+1$

$$5^{k+1} + 2 \times 11^{k+1} = 5.5^k + 2 \times 11 \times 11^k$$
$$= 5(3M - 2 \times 11^k) + 2 \times 11 \times 11^k$$
$$= 15M - 10 \times 11^k + 22 \times 11^k$$
$$= 15M + 12 \times 11^k$$
$$= 3(5M + 4 \times 11^k)$$

which is a multiple of 3.

Step 3: Since true for $n = 1$, then true for $n = 1+1$, $n = 2$ and so on for all $n \geq 1$. — 1 mark [3]

Question 13:

Marker's Comments

(a) $T = 22 + Be^{-kt}$

(i) $t=0 \quad T = 180$

$180 = 22 + Be^{0}$

$158 = B$

$T = 22 + 158e^{-kt}$

When $t = 10$, $T = 115$: $115 = 22 + 158e^{-10t}$

$\frac{93}{158} = e^{-10k}$

$k = \frac{\ln \frac{93}{158}}{-10}$

$= -\frac{1}{10} \ln \frac{93}{158}$

(ii) $T = 23$. $23 = 22 + 158e^{-kt}$

$\frac{1}{158} = e^{-kt}$

$-\frac{1}{k} \ln \frac{1}{158} = t$

$t = 95 \cdot 52$. minutes

time $= 11:30 + 1$ hr $36'$

$=$ 1:06 pm

Marker's Comments

(b) $\ddot{x} = -e^{-\frac{x}{2}}$ $\quad t=0 \; x=0 \; v=2$

(i) $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = -e^{-\frac{x}{2}}$

$\frac{1}{2}v^2 = \frac{-e^{-\frac{x}{2}}}{-\frac{1}{2}} + c$

$= 2e^{-\frac{x}{2}} + c$

$v=2$, $x=0$: $2 = 2e^{0} + c$

$0 = c$

$\therefore \frac{1}{2}v^2 = 2e^{-x/2}$

$v^2 = 4e^{-x/2}$

(ii) $v = \sqrt{4e^{-x/2}} \quad v > 0$

$= 2e^{-x/4}$

$\frac{dx}{dt} = \frac{2}{e^{x/4}}$

$\frac{dt}{dx} = \frac{e^{x/4}}{2}$

$t = \frac{1}{2} \frac{e^{x/4}}{\frac{1}{4}} + c$

$= 2e^{x/4} + c$

$t=0$, $x=0$: $0 = 2e^{0} + c$

$-2 = c$

$t = 2e^{x/4} - 2$

$\frac{t+2}{2} = e^{x/4}$

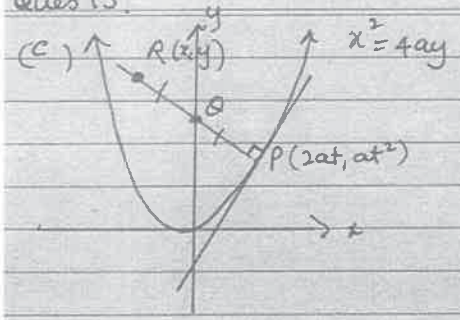
$\ln\left(\frac{t+2}{2}\right) = \frac{x}{4}$

$4 \ln\left(\frac{t+2}{2}\right) = x$

Ques 13.

Marker's Comments

(c)



(i) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

At P $x = 2at$ $\quad \frac{dy}{dx} = \frac{4at}{4a}$

$= t$

$M_T = t \quad \therefore M_N = -\frac{1}{t}$

Eqn of normal: $y - at^2 = -\frac{1}{t}(x - 2at)$

$yt - at^3 = -x + 2at$

$\underline{x + ty = 2at + at^3}$

(ii) let $x = 0$ $\quad ty = 2at + at^3$

$y = 2a + at^2$

$= a(2 + t^2)$

$\underline{Q(0, a(2+t^2))}$ (midpt of RP)

$\therefore R: \; 0 = \frac{x + 2at}{2}$ $\qquad a(2+t^2) = \frac{y + at^2}{2}$

$-2at = x$

$2a(2+t^2) = y + at^2$

$-at^2 + 4a + 2at^2 = y$

$y = at^2 + 4a$

$\underline{\therefore R(-2at, at^2 + 4a)}$

13.

Marker's Comments

(iii) Locus of R.

$x = -2at$ $\qquad y = at^2 + 4a$ ②

$\frac{x}{-2a} = t$ ①

Sub ① into ②

$y = a\left(\frac{x}{-2a}\right)^2 + 4a$

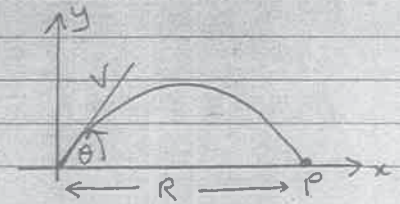
$= \frac{a x^2}{4a^2} + 4a$

$4a(y - 4a) = x^2$

$x^2 = 4a(y - 4a)$

which is a parabola with vertex $\underline{(0, 4a)}$

Question 14.



(i) $x = Vt\cos\frac{\pi}{4}$ $\qquad y = Vt\sin\frac{\pi}{4} - \frac{1}{2}gt^2$

$x = \frac{Vt}{\sqrt{2}}$ $\qquad = \frac{Vt}{\sqrt{2}} - \frac{1}{2}gt^2$ ②

$\frac{\sqrt{2}x}{V} = t$ ① ✓

Sub ① into ②

$y = \frac{V}{\sqrt{2}}\left(\frac{\sqrt{2}x}{V}\right) - \frac{1}{2}g\left(\frac{\sqrt{2}x}{V}\right)^2$

$y = x - \frac{1}{2}g \cdot \frac{2x^2}{V^2}$

$\underline{y = x - \frac{gx^2}{V^2}}$ ✓

Well done

1 mark for t

1 mark for correct substitution (which must be shown)

Marker's Comments

(ii) To find R, let $y=0$ find x — Well done.

$$0 = x - g\left(\frac{x^2}{V^2}\right)$$ — $\frac{1}{2}$ mark

$$0 = x - \frac{gx^2}{V^2}$$

$$0 = x\left(1 - \frac{gx}{V^2}\right)$$ — $\frac{1}{2}$ mark

Students must factorise x, not $\frac{1}{2}x$

$\therefore x = 0$ or $1 - \frac{gx}{V^2} = 0$

(origin)

$$1 = \frac{gx}{V^2}$$

$$\frac{V^2}{g} = x \quad \boxed{\text{but } x = R}$$

ie $R = \frac{V^2}{g}$ — 1 mark. $\boxed{2}$

Students who found t usually were successful in finding the speed.

(iii) $V = 30$ $x = 15$.

$$x = Vt\cos\theta$$

$$15 = \frac{30t}{\sqrt{2}}$$

$$\frac{15.\sqrt{2}}{30} = t$$

$$t = \frac{\sqrt{2}}{2}\text{ s}$$ — $\frac{1}{2}$ mark for t.

$\dot{x} = \frac{V}{\sqrt{2}} = \frac{30}{\sqrt{2}} = 15\sqrt{2}$ At $t = \frac{\sqrt{2}}{2}$ $\dot{y} = \frac{30}{\sqrt{2}} - 5\sqrt{2} = 15\sqrt{2} - 5\sqrt{2} = 10\sqrt{2}$

$\dot{y} = \frac{V}{\sqrt{2}} - gt$ — $\frac{1}{2}$ mark $\dot{x}$, $\frac{1}{2}$ mark $\dot{y}$ $\Big\}$ at $t = \frac{\sqrt{2}}{2}$

$$= \frac{30}{\sqrt{2}} - 10t$$ — $\frac{1}{2}$ mark.

$V = \sqrt{\dot{x}^2 + \dot{y}^2}$

$= \sqrt{\frac{900}{2} + 200}$ — $\frac{1}{2}$ mark

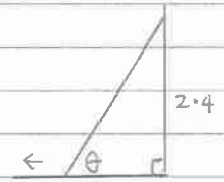
$= \sqrt{650}$

$V = \pm 25.5$ m/s

Speed $= \underline{25.5\text{ m/s}}$. — $\frac{1}{2}$ mark correct calculation $\boxed{3}$

14. Marker's Comments

(b)



$$\frac{dx}{dt} = 2\text{cm/s} = 0.02\text{ m/s}$$

$$\tan\theta = \frac{2.4}{x}$$

$$x = \frac{2.4}{\tan\theta} = 2.4(\tan\theta)^{-1}$$ — $\frac{1}{2}$ mark.

$$\frac{dx}{d\theta} = -2.4(\tan\theta)^{-2} \times \sec^2\theta$$

$$= -\frac{2.4\sec^2\theta}{\tan^2\theta}$$ — 1 mark

$$= -2.4 \cdot \frac{\cos^2\theta}{\cos^2\theta \cdot \sin^2\theta}$$

$$= \frac{-2.4}{\sin^2\theta}$$ — $\frac{1}{2}$ mark Correct simplification

OR. $x = \frac{2.4\cos\theta}{\sin\theta}$

$u = 2.4\cos\theta$
$u' = -2.4\sin\theta$
$v = \sin\theta$
$v' = \cos\theta$

$$\frac{dx}{d\theta} = \frac{vu' - uv'}{v^2}$$

$$= \frac{\sin\theta \cdot -2.4\sin\theta - 2.4\cos\theta\cos\theta}{\sin^2\theta}$$

$$= \frac{-2.4(\sin^2\theta + \cos^2\theta)}{\sin^2\theta}$$

$$= \frac{-2.4}{\sin^2\theta}$$ which is required

Marker's comments

(ii) $\frac{d\theta}{dt} = \frac{d\theta}{dx} \cdot \frac{dx}{dt}$

$= \frac{\sin^2\theta}{-2\cdot4} \times 0\cdot02$ — ½ units (2cm = 0·02m)

$= \frac{-\left(\frac{1}{\sqrt{2}}\right)^2 \times 0\cdot02}{-2\cdot4}$ — ½ substitution into correct product

$= -\frac{1}{240}$ — 1 correct answer.

$\therefore$ angle is decreasing by $\frac{1}{240}$ radians/s — $\boxed{2}$

(c) (i) $(1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1 x + {}^{2n}C_2 x^2 + \ldots\ldots + {}^{2n}C_{2n} x^{2n}$ — 1 mark. $\boxed{1}$

(ii) Differentiate b.s (i) wrt x — 1 mark

$2n(1+x)^{2n-1} = {}^{2n}C_1 + 2\,{}^{2n}C_2 x + 3\,{}^{2n}C_3 x^2 + \ldots + k\,{}^{2n}C_k x^k$
$+ \ldots + 2n\,{}^{2n}C_{2n} x^{2n-1}$

Substitute $x = -1$ — 1 mark

$0 = {}^{2n}C_1 - 2\,{}^{2n}C_2 + 3\,{}^{2n}C_3 - 4\,{}^{2n}C_4 +$
$\ldots + (-1)^n 2n.\,{}^{2n}C_{2n}$

Move the negative terms to LHS of the eqn — 1 mark

$2\,{}^{2n}C_2 + 4\,{}^{2n}C_4 + 6\,{}^{2n}C_6 + \ldots\ldots + 2n\,{}^{2n}C_{2n}$
$= {}^{2n}C_1 + 3\,{}^{2n}C_3 + 5\,{}^{2n}C_5 + \ldots + 2n-1\,{}^{2n}C_{2n-1}$

$\boxed{3}$